

Artificial Intelligence in Solving Complex Mathematical Equations: Advancing Computational Techniques and Pioneering Future Applications

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Abstract: Artificial intelligence (AI) has become a transformative force in mathematical problem solving, redefining how complex equations are formulated, analyzed, and solved. This article reviews the major advances in AI-based computational techniques, including symbolic computation, neural networks, generative models, and hybrid systems, that have enabled the handling of nonlinear, algebraic, differential, and integral equations with great efficiency and accuracy. These innovations have not only automated traditional mathematical processes, but have also enabled real-time applications in fields as diverse as physics, engineering, finance, and environmental modeling.

A detailed exploration of the methodologies highlights how AI tools, such as deep learning models, reinforcement learning algorithms, and natural language processing frameworks, have filled the gaps in traditional computer science approaches. Symbolic AI has improved equation formulation and theorem proving, while digital AI methods have improved solutions to highly nonlinear and multivariable problems. The implementation of symbolic neural systems, which combines symbolic reasoning and deep learning, once again demonstrates the versatility of AI to address higher-order mathematical challenges.

This study also examines emerging trends, such as the role of quantum computing in solving higher-dimensional equations, the adoption of generative AI to develop innovative mathematical models, and the incorporation of explainable AI (XAI) to address issues around interpretability and reliability of automated systems. The contribution of AI to educational platforms to democratize access to advanced mathematical concepts and promote new teaching methods is also examined.

Despite these advances, challenges remain, including issues of computational complexity, scalability of AI algorithms, domain-specific limitations, and ethical concerns about bias and abuse in applications. These obstacles highlight the need for collaborative efforts across disciplines to improve AI systems and connect them to the practical requirements of different industries. The article concludes with a forward-looking perspective on the potential of AI to further revolutionize mathematical problem solving. Future research directions are proposed, emphasizing the development of more robust, interpretable, and adaptive AI algorithms. The convergence of AI with emerging technologies, such as blockchain for mathematical proofs and the Internet of Things (IoT) for real-time data integration, is also discussed. This comprehensive analysis aims to provide a foundation for academics and industry practitioners to harness the full potential of AI to address mathematical complexities while navigating the challenges and ethical considerations of this rapidly evolving field.

Keywords: Artificial Intelligence, Mathematical Equations, Computational Techniques, Symbolic Reasoning.

Introduction

Artificial intelligence (AI) has become a transformative force in mathematics, offering new approaches to solving complex equations that challenge traditional computational methods. Mathematical equations, which form the foundation of many scientific and engineering disciplines, often present significant challenges due to their inherent nonlinearity, high dimensionality, and computational demands. The integration of AI, including techniques such as deep learning, symbolic computation, and hybrid models, has revolutionized this landscape, enabling researchers to solve problems previously considered intractable (Wolfram, 2020).

The effectiveness of AI in solving mathematical problems can be attributed to its ability to combine symbolic reasoning and numerical computation. Symbolic computation, characteristic of traditional algebraic methods, allows equations to be manipulated in their exact form, while the numerical capabilities of AI allow approximate solutions to complex problems where symbolic methods are insufficient (Lample and Charton, 2019). Neural networks, especially those based on deep learning architectures, have demonstrated exceptional performance in tasks such as approximating solutions to differential equations, which are essential for modeling dynamical systems in various scientific fields (Raissi, Perdikaris, & Karniadakis, 2019).

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The role of AI extends beyond computer science to the field of discovery. For example, generative models, such as variational autoencoders (VAEs) and generative adversarial networks (GANs), have been used to propose new mathematical conjectures and discover relationships between variables in large data sets (Kingma and Welling, 2014). These tools have allowed researchers to uncover hidden patterns in data that traditional mathematical methods might miss, driving innovation in fields ranging from fluid dynamics to quantum mechanics.

In addition to traditional equation processing, AI has demonstrated the ability to handle systems of equations in applied contexts. Applications include optimizing resource allocation in logistics, designing more efficient algorithms for analyzing material constraints, and even improving financial models to predict market behavior under uncertain conditions (Silver et al., 2017). In addition, AI is now being used in educational contexts, where tools such as automated theorem provers and interactive equation solvers make advanced mathematical concepts more accessible to students (Gao et al., 2021).

Despite these remarkable advances, challenges remain. One of the most pressing issues is the interpretability of AI models. Deep learning techniques, while highly effective, often function as “black boxes,” providing only solutions without a clear explanation of the underlying processes (Rudin, 2019). This lack of transparency is a significant obstacle to the adoption of AI in fields that require rigorous evidence, such as pure mathematics. Furthermore, the scalability of AI algorithms to solve equations involving millions of variables or dimensions remains a challenge, requiring further research into efficient computational techniques (Marcus, 2022).

Another critical concern is the ethical dimension of AI applications in mathematics. As AI systems become more and more autonomous, questions are raised about their potential, the fairness of algorithmic solutions, and the implications of a strong reliance on automated systems in decision-making processes. The solution of these problems will require a concerted effort by mathematicians, computer scientists and ethicists to develop frameworks for the responsible use of AI (Floridi et al., 2018). In the future, the integration of quantum computing with AI promises to solve equations of unprecedented complexity. Quantum algorithms, such as those of Shor and Grover, have already demonstrated their potential in specific mathematical domains, and their synergy with AI may pave the way for advances in higher-dimensional non-Euclidean geometry (Montanaro, 2016). Furthermore, advances in explainable AI (XAI) aim to bridge the gap between model performance and interpretability, making AI solutions more transparent and reliable in mathematical contexts (Adadi and Berrada, 2018).

This article aims to provide an in-depth exploration of the transformative role of AI in solving complex mathematical equations. By examining key advances, emerging trends, and ongoing challenges, this research aims to contribute to a growing body of knowledge and identify avenues for future innovation in this rapidly evolving field.

Literature Review

1. Introduction to AI in Mathematical Problem Solving

The integration of artificial intelligence (AI) into mathematical problem solving represents one of the most important advances in

computer science. In recent decades, AI has evolved from an emerging technology with limited applications to a sophisticated tool capable of solving extremely complex mathematical challenges. This evolution has been driven by rapid advances in symbolic computation and machine learning.

Symbolic Computation as a Foundation

Symbolic computation laid the foundation for the application of AI in mathematics. Early systems, such as Mathematica and Maple, revolutionized algebraic operations by automating functions such as symbolic differentiation, integration, and equation solving (Wolfram, 2020). These systems were powered by rule-based algorithms capable of handling precise mathematical expressions. They gave researchers the tools to manipulate equations symbolically, ensuring accuracy and reducing human error.

One of the most notable contributions of symbolic AI has been its application to theorem proving. Systems such as Prolog and automated theorem provers such as Coq and Isabelle have demonstrated the ability of AI to rigorously test mathematical proofs. These systems have become essential for checking the correctness of equations in fields such as formal methods and cryptography (Harrison et al., 2014). Despite these achievements, the reliance on predefined rules has limited the scalability of symbolic AI, especially for problems involving large or stochastic variables.

Moving to non-symbolic access

The emergence of machine learning (ML) has marked a paradigm shift in the role of AI in mathematics. Unlike symbolic systems, ML models excel in data-driven environments, where they learn patterns and relationships without relying on explicitly programmed rules. This ability makes them particularly well-suited to solving problems that defy precise symbolic representation, such as nonlinear systems, partial differential equations, and optimization problems. Lampe and Charton (2019) demonstrated the power of deep learning to solve symbolic integration problems, where neural networks were trained to mimic human problem-solving strategies. These models not only outperformed humans, but also revealed new approaches to solving equations, highlighting the potential of AI as a creative tool. In addition, reinforcement learning algorithms, such as AlphaZero, have demonstrated the ability to discover optimal strategies for iterative problem solving, providing applications in both mathematical proofs and numerical analysis (Silver et al., 2018).

Expanding capabilities with hybrid models

Hybrid approaches, which combine symbolic reasoning and machine learning, have also expanded the capabilities of AI in mathematics. For example, neural-symbolic systems use the interpretability of symbolic computation and the adaptability of neural networks to address challenges in fields such as graph theory, optimization, and mathematical modeling (Gao et al., 2021). These models have been instrumental in solving systems of equations involving discrete and continuous variables, bridging the gap between traditional mathematics and modern AI.

Practical impact across disciplines

The integration of AI into mathematical problem solving has had profound implications in various disciplines. In physics, AI-driven

solvers for the Schrödinger and Navier-Stokes equations have accelerated research in quantum mechanics and fluid dynamics (Raissi, Perdikaris, & Karniadakis, 2019). In finance, AI has been applied to solve stochastic differential equations under risk modeling and derivatives pricing (Hutchinson et al., 1994). Even in medicine, mathematical models powered by AI have been used to simulate biological processes, thus optimizing treatments for diseases such as cancer (Karpatne et al., 2017). Challenges and Opportunities

Despite its remarkable progress, AI in solving mathematical problems faces several challenges. Interpretability remains a crucial issue, as deep learning models often function as “black boxes,” providing solutions without clear explanation (Rudin, 2019). Furthermore, the scalability of AI systems for large equations and large datasets requires further research to improve computational efficiency (Marcus, 2022). However, the continued development of quantum computing and explainable AI (XAI) offers opportunities to overcome these challenges, promising to further revolutionize the field.

This synthesis of symbolic computation and machine learning has positioned AI as an indispensable tool in mathematics. By addressing theoretical and applied problems, AI continues to open up new opportunities for research and innovation, highlighting its transformative potential to solve the most complex mathematical equations.

2. UA Techniques for Solving Mathematical Equations

The application of artificial intelligence (AI) to solving mathematical equations has expanded beyond traditional computational methods to include innovative techniques that use both symbolic reasoning and data-driven approaches. These AI techniques provide new paradigms for solving complex mathematical problems, where conventional methods may be too slow, inefficient, or completely impractical. Key AI techniques in this context include symbolic AI, neural networks, hybrid approaches, and reinforcement learning, each of which offers unique capabilities for solving a variety of mathematical equations.

2.1. Symbolic UA in Mathematics

Symbolic AI uses rule-based systems to manipulate mathematical expressions by applying predefined symbols and logic, ensuring the correctness and accuracy of calculations. This approach is particularly effective in areas where precision and symbolic manipulation are essential, such as algebra, number theory, and formal verification. Symbolic AI is particularly well-suited to problems where mathematical expressions need to be simplified, differentiated, integrated, or transformed into alternative representations, with the guarantee of obtaining a mathematically valid result.

Symbolic AI systems, such as those based on logic programming (e.g., Prolog), have demonstrated effectiveness in theorem proving, where they can automatically derive new mathematical theorems from known axioms (Gao et al., 2021). Tools used in symbolic AI, such as Mathematica and Maple, can perform symbolic differentiation and integration, solve polynomial equations, and manipulate complex algebraic expressions symbolically. These systems have been essential for the advancement of fields such as cryptography, where algebraic number theory plays a key role, and in formal verification tasks,

where the correctness of mathematical models or algorithms must be rigorously confirmed. However, symbolic AI faces limitations when dealing with real, noisy, or large-scale data, as it operates on the basis of clearly defined rules and attempts to generalize beyond what is predefined. In such cases, reliance on precise representations makes symbolic AI systems inflexible in the face of ambiguities or changes in the data. This has led to a growing interest in hybridizing symbolic AI with other forms of AI, such as machine learning, to overcome these limitations and expand its applicability.

2.2. Neural Networks for Equation Approximation

Neural networks, a form of machine learning, have significantly influenced the way AI approaches solving mathematical equations, especially in contexts where symbolic methods struggle or become computationally expensive. Deep learning models, particularly neural networks, are excellent at approximating solutions to complex systems of equations, including nonlinear systems and partial differential equations (PDEs). Physics-informed neural networks (PINNs), a specialized form of deep learning, have gained particular importance in the context of solving PDEs. These models integrate physical laws (expressed as differential equations) directly into the neural network framework, allowing the network to learn solutions that adhere to fundamental physical principles, thus avoiding the need for large-scale simulations (Raissi, Perdikaris, & Karniadakis, 2019). PINNs have been used effectively in fields such as fluid dynamics, heat transfer, and electromagnetic fields, where solving PDEs is critical to understanding system behavior. The combination of data-driven learning and physical constraints makes PINNs an attractive solution for real-world engineering and scientific problems where high-fidelity simulations are computationally prohibitive.

Neural networks are particularly adept at solving high-dimensional equations, which are common in fields such as computational finance, fluid dynamics, and machine learning itself. The ability of deep learning models to generalize from data, combined with their flexibility in handling noisy or incomplete information, makes them invaluable for approximating solutions to complex equations in scenarios where analytical exact solutions do not exist or are too expensive to compute. However, challenges such as model interpretability and the risk of overfitting highlight the need for careful implementation and validation when applying neural networks to solve mathematical equations.

2.3. Hybrid approaches

Hybrid models, which integrate symbolic reasoning and machine learning techniques, represent an exciting frontier in AI-driven mathematical problem solving. These models aim to combine the strengths of symbolic AI, such as interpretability and logical rigor, with the adaptability of machine learning algorithms, which excel at handling large sets of unstructured data.

Hybrid systems are particularly useful for solving problems involving many variables, dynamic equations, or problems where different types of mathematical methods are needed at different stages of the solution process. For example, a hybrid model could use symbolic AI to initially simplify a set of equations, followed by a machine learning model to approximate the solutions numerically. This approach has been applied in fields such as optimization and control theory, where the mathematical models are very complex and involve discrete and continuous variables.

A notable example of hybrid systems in AI-driven mathematics is the use of neural-symbolic systems. These systems combine the deep learning capabilities of neural networks with the structured approach of symbolic reasoning. By combining the flexibility of machine learning with the precision of symbolic computation, these hybrid models can solve problems such as optimization in high-dimensional spaces and solve systems of equations that require symbolic manipulation and numerical approximation (Silver et al., 2017). Such integration enables more robust solutions, particularly in areas such as multivariable systems, dynamic equation solving, and large-scale mathematical models.

2.4. Reinforcement Learning in Optimization Problems

Reinforcement learning (RL) has become a powerful tool in mathematical optimization, particularly for solving nonlinear equations and optimizing complex systems. Unlike supervised learning, where the model learns from labeled data, RL algorithms learn by interacting with an environment and receiving feedback in the form of rewards or punishments. This exploration-exploitation paradigm makes reinforcement learning particularly well suited for solving optimization problems, where the solution space is large and traditional methods can be inefficient.

Reinforcement learning has found widespread use in optimization tasks such as finding the global minimum of a nonlinear function or solving combinatorial optimization problems such as the traveling salesman problem. Notable examples of RL applications include AlphaZero, a system developed by DeepMind, which has demonstrated the ability to discover new solution strategies for games, such as Go through solo play, by optimizing decision-making strategies without human intervention (Silver et al., 2018). This concept can be directly applied to solving complex mathematical equations in optimization, where RL models iteratively improve their strategies to find solutions with greater efficiency.

For mathematical equations involving high-dimensional research spaces, such as those encountered in engineering, economics, and machine learning, reinforcement learning techniques have proven effective. They allow us to explore multiple possible solutions and adapt strategies based on feedback, eventually converging to optimal or near-optimal solutions. Reinforcement learning's ability to handle complex and dynamic optimization problems marks a significant advance in the role of AI in mathematical problem solving. However, challenges remain, such as the high computational costs of training reinforcement learning models and the difficulty of adapting them to more complex, real-world scenarios.

Conclusion

AI techniques, including symbolic AI, neural networks, hybrid models, and reinforcement learning, are collectively transforming the landscape of mathematical problem solving. Symbolic artificial intelligence continues to be valuable for precise manipulation and proof validation, while neural networks, particularly in the form of PINN, have revolutionized the solution of partial differential equations. Hybrid models combining symbolic approaches and machine learning promise to address the limitations of each individual technique, and reinforcement learning pushes the limits of optimization in mathematical equations. As these techniques continue to evolve, their application to increasingly complex and dynamic mathematical problems

expands, paving the way for new advances in various scientific, engineering and industrial fields.

3. Applications of AI in Mathematical Problem Solving

The application of AI to mathematical problem solving has advanced significantly in many fields, from scientific research and engineering to finance, environmental sciences, and education. These advances in AI techniques have made it possible to solve very complex mathematical equations more efficiently, often beyond the capabilities of traditional methods. Below are some key areas where AI has demonstrated transformative potential for solving mathematical problems.

3.1. Scientific research and engineering

In scientific research and engineering, AI has become indispensable for solving complex equations that arise in fields such as fluid dynamics, structural analysis, and materials science. These equations often involve high-dimensional spaces or nonlinear systems, making traditional methods time-consuming and computationally expensive. Artificial intelligence, particularly through the use of neural networks, has shown remarkable success in solving these challenges.

One of the most notable contributions of AI to this field is its use to solve the Navier-Stokes equations, which describe the motion of incompressible fluids. Solving these equations is essential in many applications, including weather forecasting, aerospace engineering, and automotive design. Traditional computational fluid dynamics (CFD) methods can be slow and require significant computational resources. However, AI-based models, such as physics-informed neural networks (PINNs), have demonstrated the ability to efficiently solve these equations by incorporating the underlying physical laws directly into the learning process (Raissi et al., 2019). These neural network-based solvers are used in industries such as aerospace and automotive engineering, where fast and accurate simulations are essential for design and optimization. In addition, AI has facilitated advances in structural analysis by enabling real-time monitoring of infrastructure and prediction of failure points through data-driven models (Khosravi et al., 2018).

The impact of AI on materials science has also been profound. Machine learning models are used to predict material properties based on chemical composition and structural configurations, significantly accelerating the discovery of new materials (Jha et al., 2018). This is particularly useful in the development of materials for advanced technologies such as energy storage systems and semiconductor devices.

3.2. Financial Modeling and Risk Analysis

In the financial sector, AI plays a vital role in solving the equations that underpin financial models, particularly those related to option pricing and financial risk assessment. Traditional models such as Black-Scholes rely on solving stochastic differential equations to determine the price of financial derivatives. Although these models are fundamental in finance, they often make simplifying assumptions that do not capture the full complexity of financial markets.

AI techniques, particularly deep learning, have been applied to improve the accuracy of financial modeling by solving these equations more efficiently. For example, neural networks have been used to improve option pricing by learning complex patterns

in market data that traditional models cannot account for (Hutchinson et al., 1994). AI has also been used to address shortcomings in the Black-Scholes model, such as its assumptions of constant volatility and risk-free interest rates, by incorporating real-time data and dynamically adjusting the models. In addition, AI has proven to be essential in risk analysis and portfolio management. Machine learning algorithms are used to assess market risk by solving large systems of equations involving many variables, such as stock prices, interest rates, and market volatility. Reinforcement learning techniques, for example, are applied to optimize trading strategies, allowing algorithms to learn from past market data and adjust their decision-making strategies in real time (He et al., 2019). AI's ability to process complex, large-scale data makes it an indispensable tool for modern financial institutions seeking to make more informed decisions in an increasingly volatile market.

3.3. Climate Modeling and Environmental Sciences

In the field of environmental sciences, AI has become an essential tool for solving the complex equations that govern climate modeling, including those describing atmospheric dynamics, energy balance, and ocean circulation. These equations often involve high-dimensional nonlinear systems that require significant computational resources when solved using traditional numerical methods.

AI techniques, particularly deep learning and reinforcement learning, have enabled scientists to simulate and predict climate behavior more effectively. For example, AI-driven models have been used to approximate solutions to the system of equations that govern Earth's climate, which include factors such as greenhouse gas emissions, ocean temperature, and atmospheric pressure. These models can predict future climate scenarios with a level of accuracy that would be difficult or impossible to achieve with traditional methods (Karpatne et al., 2017).

In addition, AI is being used to model complex environmental processes, such as deforestation, urbanization, and water resource management. By analyzing large amounts of data from satellites, sensors, and climate models, AI algorithms can identify patterns and make predictions about future environmental trends. This ability to solve high-dimensional environmental equations in a scalable and efficient manner has important implications for policymaking and efforts to mitigate global climate change.

3.4. Educational tools

In education, AI has revolutionized the way students interact with complex mathematical concepts, making advanced mathematical tools more accessible to students at all levels. AI-powered platforms like Wolfram Alpha, Symbolab, and Mathway provide step-by-step solutions to a wide range of math problems, from algebra to calculus and differential equations. These platforms use symbolic AI and machine learning to break down problems into manageable steps and explain solutions interactively, allowing students to understand the reasoning behind each calculation (Gao et al., 2021).

Automated theorem proving and symbolic solvers reshape math pedagogy by providing students with tools to explore math concepts independently. For example, AI systems that use symbolic reasoning can prove theorems automatically, which shows how different mathematical properties and operations are related (Gao et al., 2021). This has profound implications for mathematical

research, as it can help discover new mathematical results and speed up the process of proof verification. Additionally, AI-based tutoring systems are increasingly being used in classrooms to provide personalized learning experiences. These systems analyze individual student performance, identify areas of weakness, and provide personalized exercises and explanations, allowing students to learn at their own pace. In doing so, AI is democratizing access to advanced mathematical tools and promoting deeper understanding of complex mathematical concepts that were once limited to specialized academic settings.

Conclusion

The applications of AI for solving mathematical problems are broad and varied, spanning many disciplines such as scientific research, engineering, finance, environmental science, and education. Using advanced techniques such as neural networks, hybrid models, and reinforcement learning, AI has improved the ability to solve complex mathematical equations that were previously difficult or computationally expensive. As AI continues to evolve, its role in solving mathematical challenges will only grow, providing innovative solutions across industries and contributing to advances in our understanding of the world.

4. Emerging Trends in AI for Mathematics

As artificial intelligence (AI) continues to evolve, new cutting-edge trends are emerging that are changing the way mathematics is applied and studied. These emerging trends are not only pushing the boundaries of computing power, but also introducing new methodologies for solving mathematical problems. Among them, quantum computing, generative AI, explainable AI (XAI), and interdisciplinary applications stand out as transformative forces in the mathematical problem-solving landscape. Below, we examine each of these trends in more detail, providing scientific evidence to highlight their importance and potential.

4.1. Quantum Computing and Higher Dimensional Problems

Quantum computing, combined with AI, is poised to revolutionize the solution of mathematical problems, especially in areas involving complex exponential equations. Unlike classical computers, which process information in binary (0 and 1), quantum computers use quantum bits or qubits, which can represent and process multiple states at the same time thanks to quantum superposition. This feature allows quantum computers to perform certain types of calculations at speeds unimaginable for classical systems.

The combination of quantum computing and AI has shown promise in solving problems involving high-dimensional data or difficult-to-solve optimization problems. Shor's algorithm, for example, provides an exponentially faster solution to the factorization of integers, a central problem in number theory and cryptography (Montanaro, 2016). This capability has profound implications for mathematics, especially in fields such as cryptography, where factoring large numbers is essential for securing digital communications.

Grover's algorithm, another advance in quantum computing, has demonstrated the potential to speed up search operations on unselected datasets, an approach that could be adapted to solve complex optimization problems in mathematical modeling and machine learning (Grover, 1996). By exponentially speeding up certain computations, quantum computing could pave the way for advances in areas such as combinatorial optimization, machine

learning algorithms, and large-scale simulations of physical systems governed by mathematical equations.

Although quantum computing is still in its infancy, its integration with AI may soon allow mathematicians to solve previously intractable problems, such as those involving large-scale simulations or large quantum systems, providing significant advances in both theoretical and applied mathematics.

4.2. Generative AI for Mathematical Discovery

Generative AI, which includes models such as generative adversarial networks (GANs) and major language models (e.g., GPT), is beginning to influence the way mathematical discovery is done. Traditionally, mathematical conjectures and theorems are proposed by researchers based on intuition, models, and empirical observation. However, generative AI has introduced the possibility of automating some of these processes, allowing the discovery of new mathematical relationships, patterns, and conjectures.

Generative models, such as GANs, have already been applied to areas such as image generation and data augmentation, where they create new examples of data based on learned distributions. In mathematics, a similar approach can be used to propose new mathematical structures, such as generating new conjectures or theorems by learning from a set of existing mathematical knowledge (Kingma and Welling, 2014). For example, deep learning models have been used to predict the results of complex algebraic operations or to suggest new paths in the proof of known theorems. These models can learn mathematical representations from large datasets of research papers, books, and solved problems, suggesting new hypotheses for further research.

In addition, AI-driven theorem proving is beginning to automate parts of the proof-making process. Programs like Lean and Coq, in combination with deep learning techniques, can help prove mathematical statements by suggesting intermediate lemmas and automatically checking logical steps. This has the potential to greatly accelerate the discovery of new results in pure mathematics and help mathematicians find proofs of conjectures that have remained unanswered for decades.

Thus, generative AI not only contributes to the automation of mathematical problem solving, but also promotes creativity and new directions in mathematical research, allowing mathematicians to explore a much wider solution space than was previously possible.

4.3. Explainable AI (XAI) in Mathematical Applications

While machine learning and AI have made significant progress in solving mathematical problems, one of the main challenges remains the “black box” nature of many AI models. These models, especially deep learning networks, often lack transparency, making it difficult to understand the logic behind their decisions or predictions. This is a major problem in fields where interpretability and transparency are essential, such as in scientific research, medical applications, and mathematics.

Explainable AI (XAI) seeks to address this challenge by developing methods that make AI models more transparent, interpretable, and understandable to human users (Rudin, 2019). In the context of mathematical problem solving, XAI can help researchers and practitioners understand why an AI model arrived at a certain solution, thereby ensuring that the results are valid and reliable. For example, in the application of AI to optimization

problems, XAI techniques can provide insight into how an AI system arrived at an optimal solution, explaining the factors or variables that influenced the model’s decision-making process. This is especially important in mathematics, where the validity of a solution often depends on the underlying assumptions and reasoning. XAI can also aid in mathematical modeling by revealing the relationships between variables in a model, providing a deeper understanding of the system being studied.

The growing demand for transparency in AI models underscores the importance of developing XAI techniques, especially in mathematical and scientific fields where rigorous validation is essential. As AI models become more interpretable, they are likely to be more widely accepted and applied to complex mathematical equations.

4.4. AI-driven multidisciplinary integration

One of the most exciting emerging trends is the interdisciplinary application of AI in mathematics, where AI-driven mathematical solutions are integrated across diverse fields such as biology, medicine, and economics. Mathematics serves as a unifying language across these fields, and AI’s ability to solve complex mathematical problems has led to advances in areas such as genetic modeling and the optimization of medical treatments.

In biology, AI has been used to solve mathematical equations related to genetic sequence and protein folding. Machine learning models, for example, have been used to analyze genomic data and modify the behavior of genes in response to environmental stimuli (Silver et al., 2016). By solving the equations that govern biological systems, AI helps uncover new insights into the genetics and mechanisms of disease, which can lead to new treatments and therapies. In medicine, AI is used to optimize treatment plans by solving equations that model the interactions between drugs and the human body. Reinforcement learning algorithms, for example, are used to suggest optimal treatment regimens for patients based on their individual characteristics, such as genetic information and medical history (Liu et al., 2020). This integration of AI and mathematical modeling has the potential to revolutionize personalized medicine by allowing doctors to tailor treatment plans to each patient’s unique needs.

The interdisciplinary applications of AI-driven mathematical solvers also extend to economics, where AI is used to model complex economic systems, optimize supply chains, and predict market behavior. Mathematical equations play a vital role in economic modeling, and AI techniques offer new ways to solve these equations, discover new patterns, and optimize decision-making processes (Floridi et al., 2018).

As these interdisciplinary applications continue to expand, AI-driven mathematical solvers are likely to become an essential tool in many scientific, medical, and industrial sectors, enabling researchers to solve complex, high-dimensional problems that span multiple disciplines.

Conclusion

Emerging trends in AI for mathematics, including quantum computing, generative AI, explainable AI, and interdisciplinary applications, are reshaping the way mathematical problems are approached and solved. Quantum computing promises to revolutionize the solution of highly complex problems, while generative AI models open up new avenues for mathematical discovery. Explainable AI improves the transparency and

interpretability of AI models, making them more useful for solving mathematical problems, and the interdisciplinary integration of AI is driving innovation across a wide range of fields. As these trends continue to evolve, the future of AI in mathematics looks extremely bright, with the potential to solve previously unsolved problems and drive advances across many scientific fields.

5. Challenges and limitations

Despite significant progress in AI's ability to solve complex mathematical equations, several challenges and limitations prevent its full potential. These issues include technical, ethical, and operational dimensions, which require careful attention in research and practice.

5.1. Interpretability and transparency

One of the main challenges in applying AI to solve mathematical problems is the "black box" nature of deep learning models. These systems often lack transparency, making it difficult to interpret how they arrive at solutions. This is particularly problematic in high-stakes applications such as climate modeling, financial systems, or healthcare, where rigorous validation of results is essential (Rudin, 2019). Explainable AI (XAI) initiatives aim to address these concerns, but their integration with complex mathematical solutions remains limited. For example, although symbolic AI systems offer improved interpretability, they struggle to handle nonlinear or high-dimensional equations as effectively as deep learning models. This trade-off between interpretability and computational power highlights an important gap in the current state of AI research (Doshi-Velez and Kim, 2017).

5.2. Computational Complexity

The computational requirements of AI algorithms present another important challenge. Solving large-scale systems of equations often requires significant computational resources, including high-performance hardware and significant power consumption. This limitation reduces the accessibility of AI technologies in resource-constrained environments and reduces their scalability for real-world applications (Marcus, 2022). For example, training deep learning models such as physics-informed neural networks (PINNs) for partial differential equations in climate modeling or materials science can take weeks on high-performance computer clusters. Researchers are exploring approaches such as federated learning and model optimization techniques to alleviate these challenges, but these solutions are still in the development phase (Li et al., 2020).

5.3. Ethical concerns and biases

Ethical issues in AI, including biases and the autonomy of decision-making systems, remain critical barriers to adoption. AI models are not biased unless the data they are trained on is biased. However, biased training datasets can lead to biased solutions. This is particularly concerning in fields such as finance, where biases in algorithmic results can exacerbate social inequalities (Floridi et al., 2018). Furthermore, ethical concerns extend to the autonomy of AI systems in solving mathematical problems. The risk of over-reliance on AI-generated solutions, without adequate human oversight, raises questions about the accountability and reliability of decision-making processes (Jobin et al., 2019). Regulatory frameworks are needed to ensure ethical practices in the implementation of AI, but their development often lags behind the pace of technological advances.

5.4. Domain-Specific Challenges

AI systems also face challenges specific to certain application domains. For example, in scientific research, integrating AI with traditional mathematical tools often requires domain expertise, which creates a barrier to adoption. In educational contexts, AI-based tools are underutilized due to a lack of teacher training and curriculum integration (Gao et al., 2021). These domain-specific barriers highlight the need for interdisciplinary collaboration to develop tailored AI solutions that meet the unique requirements of each application domain.

5.5. Credibility in high-stakes scenarios

In high-stakes scenarios such as aerospace engineering or medical diagnostics, the reliability of AI systems is essential. The inherent uncertainties in deep learning models, along with the lack of rigorous error bounds in many AI solutions, compromise their adoption in such contexts. Research on quantifying uncertainty and robustness of AI-driven mathematical solvers is gaining ground, but these methods are not yet mature (Smith and Gal, 2018).

Conclusion

Addressing these challenges is essential to advancing the role of AI in solving mathematical problems. Future research should prioritize the development of scalable, interpretable, and ethically sound AI models, fostering interdisciplinary collaboration to overcome domain-specific barriers. By addressing these issues, AI can become a reliable and transformative tool in various areas of mathematics and its applications.

6. Summary of gaps in the literature

Despite advances in artificial intelligence (AI) for solving mathematical problems, significant gaps remain in the existing literature. These gaps highlight areas where more research is needed to overcome current limitations and unlock the full potential of AI in mathematical applications. Addressing these gaps will require a concerted effort across multiple disciplines, including theoretical, computational, and ethical considerations.

6.1. Lack of interpretability in AI models

One of the most pressing gaps in the literature is the limited interpretability of AI models used to solve mathematical equations. Although machine learning models, particularly deep neural networks, have demonstrated remarkable ability to approximate solutions to complex systems, their "black box" nature makes it difficult to understand how these solutions are derived (Rudin, 2019). This lack of transparency is particularly problematic in applications that require rigorous mathematical testing, such as scientific research and engineering. Existing studies have focused primarily on improving model accuracy, often at the expense of interpretability (Lipton, 2018). There is a critical need for research that explores methods to improve the explainability of AI models without compromising their performance. Techniques such as interpretable AI (XAI), interpretable neural networks, and symbolic regression hold promise but remain unexplored in the context of solving complex mathematical problems.

6.2. Scalability for large and large-scale systems

AI models face significant challenges when applied to large-scale systems of equations or problems of large dimensions. Current

methods often struggle to be computationally efficient and scalable, limiting their applicability to real-world scenarios such as climate modeling, financial forecasting, and multi-body physics simulation (Marcus, 2022).

Although advances in hardware, such as graphics processing units (GPUs) and tensor processing units (TPUs), have improved computational capabilities, there is a lack of algorithmic innovations suitable for large-scale mathematical systems. Research is needed to develop more efficient AI algorithms that can handle the computational demands of large datasets and high-dimensional equations without sacrificing accuracy or precision.

6.3. Limited integration of symbolic and digital approaches

Another obvious gap is the limited integration of symbolic and numerical approaches in AI-based mathematical problem solving. Symbolic AI excels at exact computations and theorem proving, while digital AI, particularly neural networks, is better suited to approximate solutions to complex systems. However, hybrid models that combine the strengths of both approaches remain underdeveloped (Silver et al., 2017).

The existing literature mainly treats symbolic and digital methods as separate fields, and few studies explore their synergies. The development of hybrid frameworks that seamlessly integrate symbolic reasoning and numerical approximation can fill this gap, allowing AI systems to address a wider range of mathematical problems with greater efficiency and accuracy.

6.4. Lack of interdisciplinary frameworks

The application of AI to mathematics often requires interdisciplinary collaboration because mathematical problems are closely related to fields such as physics, biology, economics, and engineering. However, the literature reveals a lack of comprehensive frameworks that integrate mathematical AI with domain-specific knowledge. For example, although AI has shown promise in solving partial differential equations in fluid dynamics or epidemiological models, these applications often lack a strong connection between mathematical algorithms and the domain-specific principles governing the systems (Raissi et al., 2019).

Future research should focus on developing interdisciplinary frameworks that combine AI-driven mathematical solvers with knowledge from other scientific fields. This would facilitate more accurate modeling of real-world phenomena and improve the applicability of AI to solve complex, domain-specific problems.

6.5. Ethical and Social Implications

Although the technical challenges of AI in solving mathematical problems are well documented, research on the ethical and social implications of these technologies is limited. Biases in training data, liability for AI-generated solutions, and the potential for AI to be used in sensitive applications remain underexplored in the context of mathematics (Floridi et al., 2018).

For example, AI systems used in financial modeling or public policy often rely on mathematical equations to make predictions or inform decisions. If these systems are biased or lack transparency, they can lead to inconsistent results. Addressing these ethical considerations will require a multidisciplinary approach that combines technical advances with rigorous ethical frameworks.

6.6. Limited focus on educational applications

AI-powered mathematical tools, such as automated theorem provers and solvers, have the potential to revolutionize education by making advanced mathematical concepts more accessible. However, the literature shows limited attention to the educational implications of these tools (Gao et al., 2021). Most studies focus on the technical capabilities of AI systems and pay little attention to their impact on learning outcomes or their integration into school curricula.

Future research should explore how AI tools can be designed to support active learning, foster mathematical intuition, and respond to the diverse needs of students. This includes studying the effectiveness of AI systems in promoting conceptual understanding, problem-solving skills, and critical thinking.

6.7. Emerging Areas of Mathematical AI

Finally, there is a lack of exploration into emerging areas where AI can play a transformative role in mathematics. These include quantum computing for solving high-dimensional problems, generative AI for discovering new mathematical relationships, and reinforcement learning for optimization problems (Montanaro, 2016; Silver et al., 2018). Although these areas are promising, the existing literature provides limited evidence of their practical applications or scalability to real-world problems.

Conclusion

Addressing these gaps in the literature will be critical to advancing the application of AI to mathematical problem solving. Future research should prioritize developing interpretable and scalable models, fostering interdisciplinary collaborations, addressing ethical issues, and exploring emerging areas of mathematical AI. By addressing these challenges, the field can move closer to realizing the full potential of AI as a transformative tool for mathematics and its applications across diverse fields.

Methodology

The methodology used in this study involves a mixed method approach, combining a qualitative analysis of the existing literature with a quantitative evaluation of AI models in solving mathematical problems. The research focuses on examining and synthesizing key developments, trends and challenges in the application of AI to solve complex mathematical equations. The following steps describe the methodological framework:

1. Literature review and data collection

A comprehensive review of academic articles, technical reports, and case studies was conducted to identify advances, techniques, applications, and challenges in AI-driven mathematical problem solving. Databases such as IEEE Xplore, SpringerLink, and Google Scholar were used to collect peer-reviewed resources. Emphasis was placed on seminal work, such as the fundamental theories of symbolic AI, and recent innovations, including neural networks and hybrid models.

2. Qualitative analysis

The qualitative aspect of the methodology includes thematic coding and literature analysis to identify recurring trends, emerging technologies, and theoretical underpinnings of AI applications in mathematics. This step also includes an assessment

of ethical issues, limitations, and interpretation challenges associated with AI models.

3. Quantitative evaluation of AI techniques

Quantitative data comes from existing research studies that have evaluated the performance of AI techniques in solving specific types of mathematical problems:

Symbolic AI: Success rates in theorem proving, equation simplification, and symbolic differentiation.

Neural networks: Accuracy and computational efficiency of physics-based neural networks (PINNs) in solving partial differential equations (PDEs).

Reinforcement learning: The performance of iterative optimization in solving nonlinear equations.

Performance measures such as computational time, accuracy, and scalability were compared between different methods.

4. Comparative Analysis

A comparative analysis was conducted to assess the effectiveness of symbolic AI, digital AI, hybrid approaches, and reinforcement learning in various fields, such as engineering, climate modeling, and financial mathematics. Particular attention was paid to emerging technologies, such as quantum computing and generative AI, to explore their transformative potential.

5. Identifying gaps and trends

The synthesis of qualitative and quantitative results helped identify outstanding challenges and emerging opportunities in AI-based mathematical problem solving. These findings informed recommendations for future research directions and applications.

6. Evaluation through case studies

Case studies from various fields, such as fluid dynamics, financial modeling, and education, were examined to validate theoretical understanding and illustrate the practical application of AI in solving mathematical equations. These real-world examples provided evidence of the feasibility and limitations of various AI techniques.

Ethical considerations

The study followed ethical research practices, ensuring transparency and objectivity in the evaluation of AI systems. Particular attention was paid to accounting for biases in training data, accountability for solutions generated by AI, and the ethical implications of implementing AI in sensitive mathematical applications. This blended approach provided a robust framework for exploring the advances, applications, and challenges of AI in solving mathematical problems, ensuring that the results are comprehensive and evidence-based.

Theoretical Framework

This study relies on several theories to explain the application of artificial intelligence (AI) to solve complex mathematical equations. These theories provide the basis for understanding how AI techniques are developed, applied, and evaluated to solve mathematical problems. Among them, the main guiding theory is the theory of computational learning, which supports the development of machine learning algorithms and their application to solve mathematical problems.

1. Theory of computer learning (theory of instruction)

Computational learning theory, a subfield of theoretical computer science, studies the design and analysis of algorithms for learning from data. It provides a framework for understanding how AI models, particularly machine learning algorithms, can generalize patterns from input data to solve mathematical equations. This theory has guided research with an emphasis on:

Learning: how AI models gain the ability to approximate mathematical solutions.

Generalization: the ability of AI systems to apply learned models to unseen problems.

Algorithm complexity: understanding the trade-off between computational efficiency and model accuracy.

For example, the use of neural networks to solve partial differential equations (PDEs) relies heavily on principles of computer learning theory, such as loss function optimization and convergence analysis (Goodfellow et al., 2016).

2. Symbolic theory of AI

The symbolic theory of AI, rooted in logical and rule-based systems, forms the basis for the use of AI in precise mathematical calculations. This theory focuses on how predefined rules and symbolic representations can be used for tasks such as theorem proving, equation simplification, and symbolic differentiation. He led the exploration of symbolic solvers such as systems and Prolog-based tools such as Mathematica (Gao et al., 2021).

3. Neural Network Theory

Neural network theory, based on artificial neural networks (ANNs), explains how data-driven models can approximate complex mathematical functions. The study applied this theory to evaluate the performance of physics-based neural networks (PINNs) and other deep learning models in solving nonlinear and large-dimensional systems of equations (Raissi et al., 2019).

4. Reinforcement Learning Theory

Reinforcement learning (RL) theory focuses on how agents learn optimal strategies through exploration and exploitation in dynamic environments. This theory has been instrumental in analyzing iterative methods for optimization problems and solving nonlinear equations. The AlphaZero algorithm, for example, illustrates the power of reinforcement learning in discovering new solution strategies (Silver et al., 2018).

5. Hybrid AI Theory

Hybrid AI theory integrates symbolic and numerical approaches, emphasizing the synergies between rule-based reasoning and data-driven learning. This theory has supported the evaluation of hybrid models that exploit the interpretability of symbolic AI and the adaptability of machine learning, enabling solutions for complex multivariate systems (Silver et al., 2017).

6. Explainable AI Theory (XAI).

Explainable AI theory addresses the interpretability and transparency of machine learning models. It has driven research on how to make mathematical solutions generated by AI understandable and verifiable, especially in critical applications such as scientific research and finance (Rudin, 2019).

The rationale for computational learning theory as a major instructional theory

Computational learning theory was chosen as the main guiding theory because it incorporates principles from symbolic and digital AI, bridging the gap between rule-based systems and data-driven models. Its focus on learning, generalization, and computational efficiency aligns closely with the goals of the study, which include assessing AI's ability to solve mathematical problems, identifying limitations, and proposing future directions. This theory provides a comprehensive framework for understanding the challenges and opportunities of applying AI to mathematical problem solving, making it more relevant to this research.

Discussion

The application of artificial intelligence (AI) to solving complex mathematical equations has opened new avenues for mathematical research and applications in various fields. This discussion summarizes the findings of the study, highlights the progress, emerging trends, ongoing challenges, and implications for future research and practice.

AI techniques, such as symbolic AI, neural networks, and hybrid models, have shown tremendous potential for solving mathematical problems. Symbolic AI has excelled at tasks that require precise solutions, such as theorem proving and algebraic manipulations, using tools such as Mathematica and Prolog-based systems. Neural networks, especially physics-informed neural networks (PINNs), have revolutionized the numerical approach to solving complex partial differential equations that underlie fields such as fluid dynamics and climate modeling. Hybrid approaches, which integrate symbolic reasoning and data-driven learning, have proven effective in dealing with multivariable and dynamic systems, combining interpretability and adaptability.

Emerging trends in AI offer exciting opportunities for mathematical innovation. Quantum computing has the potential to solve complex exponential equations, addressing challenges that traditional computing methods cannot address. Generative AI models have been explored for mathematical discovery, providing opportunities for automated theorem proving and the formulation of new conjectures. Explainable AI (XAI) addresses the critical need for transparency, making AI solutions interpretable and verifiable, especially in high-stakes applications such as scientific research and finance. These advances highlight the role of AI as a transformative tool in advancing mathematical discovery and interdisciplinary research.

However, significant challenges remain to unlock the full potential of AI in mathematics. The "black box" nature of deep learning models often limits their interpretability, which presents problems in theoretical applications where validation and accuracy are paramount. Computational complexity and scalability issues hinder AI's ability to efficiently handle large-scale systems of equations, thereby reducing its applicability in real-world scenarios. Ethical issues, such as bias in training data and AI autonomy in decision-making, also complicate its adoption in critical areas. These challenges require a focused effort to develop interpretable, scalable, and ethical AI systems.

The study also identifies gaps in existing research, including the need for models that balance computational efficiency with interpretability and frameworks that facilitate interdisciplinary

applications of AI in mathematics. Educational applications remain unexplored, with considerable potential to democratize access to advanced mathematical tools and improve learning outcomes. Bridging these gaps requires a multidisciplinary approach that integrates advances in AI with domain-specific knowledge and pedagogy.

In conclusion, AI has made great progress in solving mathematical equations, but its potential remains partially exploited. Integrating symbolic and digital methods, advances in emerging technologies, and attention to ethical and practical considerations will be essential for future progress. By addressing these challenges and filling these gaps, AI can become an indispensable tool for mathematical innovation, driving advances in science, engineering, education, and beyond.

Conclusion

The integration of artificial intelligence (AI) into the solution of complex mathematical equations marks a significant advance in computer science, enabling advances in fields as diverse as engineering, finance, climate science, and education. This study explores the current capabilities, emerging trends, challenges, and future directions of AI-driven mathematical problem solving, providing a comprehensive analysis of its transformative potential.

AI techniques have evolved from symbolic AI systems, which rely on predefined rules for precise algebraic operations, to neural networks that approximate solutions to complex, nonlinear equations. Hybrid approaches, which combine symbolic reasoning with machine learning and optimization techniques based on reinforcement learning, have further expanded the applicability of AI. These advances highlight AI's ability to solve problems that traditional digital methods often fail to address, especially in high-dimensional and dynamic systems. Emerging trends, including quantum computing, generative AI, and explainable AI (XAI), are reshaping the landscape of AI in mathematics. Quantum computing provides solutions to exponential complexity equations, generative AI facilitates mathematical discovery by automating proofs and conjecture, and XAI addresses the need for interpretability in AI models. These trends highlight the increasingly interdisciplinary nature of AI-driven mathematics, with applications ranging from scientific research to genetic modeling.

Despite these advances, significant challenges remain. The "black box" nature of AI models raises concerns about their interpretability and transparency, especially in critical applications that require rigorous validation. Computational complexity and resource requirements limit scalability, while ethical concerns about bias and autonomy require careful monitoring. Furthermore, research gaps, such as the lack of scalable and interpretable models and unexplored educational applications, need to be addressed to unlock the full potential of AI in mathematics.

This study highlights the importance of computational learning theory as a guiding framework, supported by other theories such as symbolic AI, neural network theory, and explainable AI. The results highlight the need for future research to focus on developing hybrid models that combine accuracy and adaptability, fostering interdisciplinary collaboration and prioritizing ethical and practical considerations in AI implementation.

In summary, although AI has significantly advanced mathematical problem solving, its transformative potential has yet to be fully realized. By addressing challenges and filling research gaps, AI can become an indispensable tool for innovation, driving advances in theoretical mathematics, applied sciences, and global education. This study serves as a roadmap for future exploration and application of AI in mathematics, with the goal of maximizing its impact across disciplines.

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