

Systems of Coupled Stochastic Differential Equations with Piecewise Initial Conditions for Access and Fidelity Bank Share Prices

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Abstract — This system models two correlated bank stocks subject to a regulatory regime shift at time t , 2-dimensional correlated Geometric Brownian Motion with piecewise constant Initial conditions is used to represent policy announcements such as Central Bank of Nigeria (CBN) recapitalization. The Stochastic Differential Equations are coupled via correlated Brownian Motion (BM) with correlated ρ . The model demonstrates that regulatory shocks create short-term alignment but long-term divergence. For investors and regulators, it is critical to distinguish between the effect of policy and idiosyncratic performance when forecasting the future share prices of Access Bank and Fidelity Bank. Finally the existence and uniqueness of a strong solution on each regime interval is established. The model is applied to evaluate event-driven trading and systemic risk.

Keywords: Asset Value Prices, Regime-Switching, Stochastic Analysis, CBN and Brownian Motion.

1.1 Introduction

The Nigerian banking sector is highly interconnected, such that a single regulatory or macroeconomic event often affects multiple banks simultaneously. To capture this joint behavior, a system of coupled Stochastic Differential Equations (SDEs) is required in particular, Access Bank and Fidelity Bank are exposed to common market factors but also retain bank-specific risks. To reflect regulatory interventions such as Central Bank of Nigeria (CBN) recapitalization directives, the model incorporates piecewise deterministic initial conditions. At the intervention time, the initial jumps from $S_{0,1}^{(i)}$ to $S_{0,2}^{(i)}$, representing an instantaneous revaluation of both banks. The use of piecewise initial conditions allows the model to separate the immediate policy effect from the subsequent stochastic evolution driven by $\sigma_i W_t^{(i,c)}$. This framework is particularly relevant for Access and Fidelity because it enables analysis of three issues: post-shock path divergence, the impact of correlation on systemic risk and portfolio diversification. Therefore, the coupled SDE system with piecewise initial conditions provides a realistic tool for forecasting and risk assessment of Access and Fidelity Bank share prices in an event-driven market environment. Therefore, it is important to understand properly the type of physical quantities to

model so that realistic results of the stock variables can be obtained, see [1].

Hence, Geometric Brownian Motion (GBM) has been widely applied for short-term stock price forecasting due to its analytical tractability [2-3].

However, a lot of scholars has worked on SDE problems, for instance, [4] looked at the stochastic model of price changes on the floor of stock market. [5] studied the stochastic analysis of stock market expected returns and growth-rates. The precise conditions for obtaining the drifts, volatilities, Growth-Rates of four different assets were obtained. In a similar manner, [6] studied on the stochastic model of the changes or variations of stock market prices. Conditions for finding out the equilibrium price, adequate circumstances for dynamic stability and convergence to equilibrium of the growth rate of the value function of shares. Besides, [7] studied the unstable nature of stock market forces using proposed differential equation model. Also [8] looked at stochastic analysis of the behaviour of stock prices. Results show that the proposed model is efficient for the prediction of stock prices. [9] worked on the solution of differential equations and stochastic differential equations of time-varying investment returns; where precise conditions were obtained which governs stock return rates through multiplicative effect and multiplicative inverse

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trends series. [10] considered the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE), in this study, the drift and volatility coefficients for the stochastic differential equations were determined. [11] worked on the problem of stock price fluctuations using stochastic differential equations. Details of the analytical solutions were given; the computational and graphical results were presented and discussed. [12] considered the stochastic analysis of Markov Chain in Finite state. The transition Matrix replicated the use of 3-State transition probability matrix which enables them to present precise condition of obtaining expected mean rate of returns of each stock.

Nevertheless, policy announcements cause discontinuous jumps not captured by standard GBM. Existing studies treat policy as exogenous, without modeling the jump in $s_0(t)$. This paper aims to quantify the impact of regulatory shocks on correlated bank stocks and to evaluate event-driven trading strategies. Hence, this paper compliments the works of [13] and [14] respectively. This study models Access and Fidelity share prices using a 2-dimensional coupled GBM with correlation between their Brownian motions. There is no SDE framework for CBN recapitalization impact on multiple banks with correlation which have not been seen elsewhere.

The arrangement of this paper is set as follows: Section 2.1 presents the mathematical framework, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

2.1: Mathematical Framework

At this stage we present few intricate definitions touching this dynamic area of study, hence we have the following:

Definition 1: Probability space is a triple $(\Omega, \mathcal{F}, \mathbb{P})$ where Ω represents a set of sample space, $\mathcal{F}$ represents a collection of subsets of Ω , while $\mathbb{P}$ is the probability measure defined on each event $A \in \mathcal{F}$. The collection $\mathcal{F}$ is a σ -algebra or σ -field such as $\Omega \in \mathcal{F}$ and $\mathcal{F}$ is closed under the arbitrary unions and finite intersections. Hence it is called probability measure when the following condition holds.

- (i) $P(A) \geq 0$ for all $A \subset \Omega$
- (ii) $P(\Omega) = 1$
- (iii) $A, B \subset \Omega, A \cap B = \emptyset$ then $P(A \cup B) = P(A) + P(B)$

Definition 2: A σ -algebra is a set $\mathcal{F}$ of subsets of Ω with the following axioms:

- (i) $\emptyset, \Omega \in \mathcal{F}$
- (ii) If $A \in \mathcal{F}$, then $A^c \in \mathcal{F}$

- (iii) If $A_1, A_2, \dots, \in \mathcal{F}$, then $\bigcup_{k=1}^{\infty} A_k, \bigcap_{k=1}^{\infty} A_k \in \mathcal{F}$

Clearly $A^c := \Omega - A$ is the complement of A , see [19-20].

Definition 3: A stochastic process whose finite dimensional probability distributions are all Gaussian. (Normal distribution).

where X_k are independent and identically distributed random variables.

Therefore μ is not known completely which is subject to environmental effects.

Therefore the stock dynamics governing this process can be written as:

$$dS(t) = \mu S(t) dt + \sigma S(t) dZ(t)$$

Where σ is the volatility, dZ is the Brownian motion which is random term, [18].

Definition 4: A Stochastic Differential Equation is a differential equation with stochastic term. Therefore assume that $(\Omega, \mathcal{F}, \mathbb{P})$ is a probability space with filtration $\{\mathcal{F}_t\}_t \geq 0$ and $W(t) = (W_1(t), W_2(t), \dots, W_m(t))^T, t \geq 0$ an m -dimensional Brownian motion on the given probability space. We have SDE in coefficient functions of f and g as follows

$$dX(t) = f(t, X(t))dt + g(t, X(t))dZ(t), 0 \leq t \leq T,$$

$$X(0) = x_0,$$

where $T > 0$, x_0 is an n -dimensional random variable and coefficient functions are in the form $f: [0, T] \times \mathbb{R}^n$ and $g: [0, T] \times \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$. SDE can also be written in the form of integral as follows:

$$X(t) = x_0 + \int_0^t f(S, X(S))dS + \int_0^t g(S, X(S))dZ(S)$$

Where dX, dZ are known as stochastic differentials. The $\mathbb{R}^n$ is a valued stochastic process $X(t)$, [15-17].

Theorem 1.1: (Ito's lemma). Let $f(S, t)$ be a twice continuous differential function on $[0, \infty) \times \mathbb{A}$ and let S_t denotes an Ito's process

$$dS_t = a_t dt + b_t dz(t), t \geq 0,$$

Applying Taylor series expansion of F gives:

$$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} (dS_t)^2 + \text{higher order terms (h.o.t)}$$

,

So, ignoring h.o.t and substituting for dS_t we obtain

$$\begin{aligned} dF_t &= \frac{\partial F}{\partial S_t} (a_t dt + b dz(t)) + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} (a_t dt + b dz(t))^2 \\ &= \frac{\partial F}{\partial S_t} (a_t dt + b dz(t)) + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} b^2 dt, \\ &= \left(\frac{\partial F}{\partial S_t} a_t + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} b^2 \right) dt + \frac{\partial F}{\partial S_t} b_t dz(t) \end{aligned}$$

More so, given the variable $S(t)$ denotes stock price, then following GBM implies, the function $F(S, t)$, Ito's lemma gives:

$$dF = \left(\mu S \frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 F}{\partial S^2} \right) dt + \sigma S \frac{\partial F}{\partial S} dz(t)$$

2.1.1 Problem Formulation

Here, we model the joint dynamics of Access Bank and Fidelity Bank subject to a Central Bank of Nigeria recapitalization policy shock at $t = 1$ year.

Therefore, we model the stock.

$$dS_t^{(1)} = \mu_1 S_t^{(1)} dt + \sigma_1 S_t^{(1)} dW_t^{(1)} \quad (1.1)$$

$$dS_t^{(2)} = \mu_2 S_t^{(2)} dt + \sigma_2 S_t^{(2)} dW_t^{(2)} \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) \quad (1.2)$$

$$S_0^{(1)}(t) = \begin{cases} S_0', t < 1 \\ S_0'', t \geq 1 \end{cases}, \quad S_0^{(2)}(t) = \begin{cases} S_0^*, t < 1 \\ S_0^{**}, t \geq 1 \end{cases} \quad (1.3)$$

where S_0', S_0'' represents independent initial prices of Access and Fidelity, S_0^*, S_0^{**} represents independent initial prices of Access and Fidelity k represent growth rate, μ_1 represents drift coefficient of Access Bank, σ_1 represents volatility coefficient of Access Bank, μ_2 represents drift coefficient of Fidelity Bank, σ_2 represents volatility coefficient of Fidelity Bank $W_t^{(1)}, W_t^{(2)}$ represents independent Brownian Motions and t represents time.

2.1.2 Method of Solution

The investment equation (1.1)-(1.3) consist of a system of variable coefficient problem whose solutions are not trivial.

We apply Theorem 1.1 in solving for $S_t^{(1)}$, and $S_t^{(2)}$. let

$f(S_t^{(1)}, t) = \ln S_t^{(1)}$ and taking the partial derivatives yields:

$$\left\{ \frac{\partial f}{\partial S_t^{(1)}} = \frac{1}{S_t^{(1)}}, \frac{\partial^2 f}{\partial S_t^{(1)2}} = -\frac{1}{S_t^{(1)2}}, \frac{\partial f}{\partial t} = 0 \right\} \quad (1.4)$$

According to Ito's theorem gives:

$$df(S_t^{(1)}, t) = \sigma_1 S_t^{(1)} \frac{\partial f}{\partial S_t^{(1)}} dW_t^{(1)} + \left(\mu_1 S_t^{(1)} \frac{\partial f}{\partial S_t^{(1)}} + \frac{1}{2} \sigma_1^2 S_t^{(1)2} \frac{\partial^2 f}{\partial S_t^{(1)2}} \right) dt \quad (1.5)$$

Substituting (1.4) into (1.5) gives the following:

$$df(S_t^{(1)}, t) = \sigma_1 S_t^{(1)} \frac{1}{S_t^{(1)}} dW_t^{(1)} + \left(\mu_1 S_t^{(1)} \frac{1}{S_t^{(1)}} + \frac{1}{2} \sigma_1^2 S_t^{(1)2} \left(-\frac{1}{S_t^{(1)2}} \right) + 0 \right) dt \quad (1.6)$$

simplifying gives

$$= \sigma_1 S_t^{(1)} \frac{S_t^{(1)}}{S_t^{(1)}} dW_t^{(1)} + \left(\mu_1 \frac{S_t^{(1)}}{S_t^{(1)}} - \frac{1}{2} \sigma_1^2 \frac{S_t^{(1)2}}{S_t^{(1)2}} \right) dt \quad (1.7)$$

$$= \sigma_1 S_t^{(1)} dW_t^{(1)} + \left(\mu_1 - \frac{1}{2} \sigma_1^2 \right) dt$$

$$df(S_t^{(1)}, t) = \left(\mu_1 - \frac{1}{2} \sigma_1^2 \right) dt + \sigma_1 dW_t^{(1)} \quad (1.8)$$

Integrating (1.8) from 0 to t gives

$$\int_0^t d \ln S_t^{(1)} = \int_0^t df(S_t, u) = \int_0^t \left(\mu_1 - \frac{1}{2} \sigma_1^2 \right) du + \int_0^t \sigma_1 dW_t^{(1)} du \quad (1.9)$$

$$\ln S_t^{(1)} - \ln S_t^{(1)} = \left(\mu_1 u - \frac{1}{2} \sigma_1^2 u \right)_0^t + \left(\sigma_1 dW_t^{(1)} u \right)_0^t$$

$$\ln \left(\frac{S_t^{(1)}}{S_0^{(1)}} \right) = \left(\mu_1 - \frac{1}{2} \sigma_1^2 \right) t + \sigma_1 dW_t^{(1)}$$

Taking the ln of the both sides gives

$$S_t^{(1)} = S_0^{(1)} \exp \left(\left(\mu_1 - \frac{1}{2} \sigma_1^2 \right) t + \sigma_1 dW_t^{(1)} \right) \quad (1.10)$$

This gives the general solution of the SDE. To find the particular solution

$$S_0^{(1)}(t) = S'_0 \quad ,$$

$$\Rightarrow S_0^{(1)} \exp\left(\mu_1 - \frac{1}{2}\sigma_1^2\right)t + \sigma_1 dW_t^{(1)} = S'_0$$

$$S_0^{(1)} = S'_0$$

$$S_0^{(1)}(t) = S'_0 \exp\left(\left(\mu_1 - \frac{1}{2}\sigma_1^2\right)t + \sigma_1 dW_t^{(1)}\right) \quad (1.11)$$

Also,

$$S_0^{(1)}(t) = S''_0 \quad ,$$

$$\Rightarrow S_0^{(1)} \exp\left(\mu_1 - \frac{1}{2}\sigma_1^2\right)t + \sigma_1 dW_t^{(1)} = S''_0$$

$$S_0^{(1)} = S''_0$$

$$S_0^{(1)}(t) = S''_0 \exp\left(\left(\mu_1 - \frac{1}{2}\sigma_1^2\right)t + \sigma_1 dW_t^{(1)}\right) \quad (1.12)$$

Combining (1.11) and (1.12) gives the following equations

$$S_0^{(1)}(t) = \begin{cases} S'_0 \exp\left(\left(\mu_1 - \frac{1}{2}\sigma_1^2\right)t + \sigma_1 dW_t^{(1)}\right), t < 1 \\ S''_0 \exp\left(\left(\mu_1 - \frac{1}{2}\sigma_1^2\right)t + \sigma_1 dW_t^{(1)}\right), t \geq 1 \end{cases}$$

From (1.2), let $f(S_t^{(2)}, t) = \ln S_t^{(2)}$ and taking the partial derivatives yields:

$$\left. \frac{\partial f}{\partial S_t^{(2)}} = \frac{1}{S_t^{(2)}}, \frac{\partial^2 f}{\partial S_t^{(2)2}} = -\frac{1}{S_t^{(2)2}}, \frac{\partial f}{\partial t} = 0 \right\} \quad (1.14)$$

According to Ito's theorem gives:

$$df(S_t^{(2)}, t) = \sigma_2 S_t^{(2)} \frac{\partial f}{\partial S_t^{(2)}} \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) + \left(\mu_2 S_t^{(2)} \frac{\partial f}{\partial S_t^{(2)}} + \frac{1}{2} \sigma_2^2 S_t^{(2)2} \frac{\partial^2 f}{\partial S_t^{(2)2}} \right) dt \quad (1.15)$$

Substituting (1.14) into (1.15) gives the following:

$$df(S_t^{(2)}, t) = \sigma_2 S_t^{(2)} \frac{1}{S_t^{(2)}} \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) + \left(\mu_2 S_t^{(2)} \frac{1}{S_t^{(2)}} + \frac{1}{2} \sigma_2^2 S_t^{(2)2} \left(-\frac{1}{S_t^{(2)2}} \right) + 0 \right) dt \quad (1.16)$$

simplifying (1.16) gives

$$= \sigma_2 S_t^{(2)} \frac{S_t^{(2)}}{S_t^{(2)}} \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) + \left(\mu_2 \frac{S_t^{(2)}}{S_t^{(2)}} - \frac{1}{2} \sigma_2^2 \frac{S_t^{(2)2}}{S_t^{(2)2}} \right) dt \quad (1.17)$$

$$= \sigma_2 S_t^{(2)} \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) + \left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) dt$$

$$df(S_t^{(2)}, t) = \left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) dt + \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) dW_t \quad (1.18)$$

Integrating (1.19) from 0 to t gives

$$\int_0^t d \ln S_t^{(2)} = \int_0^t df(S_t^{(2)}, u) = \int_0^t \left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) du + \int_0^t \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) du \quad (1.19)$$

$$\ln S_t^{(2)} - \ln S_0^{(2)} = \left(\mu_2 u - \frac{1}{2} \sigma_2^2 u \right)_0^t + \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) dW_u \Big|_0^t$$

$$\ln \left(\frac{S_t^{(2)}}{S_0^{(2)}} \right) = \left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) t + \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right)$$

Taking the ln of the both sides gives

$$S_t^{(2)} = S_0^{(2)} \exp \left(\left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) t + \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) \right) \quad (1.20)$$

This gives the general solution of the SDE. To find the particular solution

$$S_0^{(2)}(t) = S_0^*$$

$$\Rightarrow S_0^{(2)} \exp \left(\left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) t + \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) \right) = S_0^*$$

$$S_0^{(2)} = S_0^* \quad (1.13)$$

$$S_0^{(2)}(t) = S_0^* \exp \left(\left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) t + \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) \right) \quad (1.21)$$

Also,

$$S_0^{(2)}(t) = S_0^{**}$$

$$\Rightarrow S_0^{(2)} \exp \left(\left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) t + \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) \right) = S_0^{**}$$

$$S_0^{(2)} = S_0^{**}$$

$$S_0^{(2)}(t) = S_0^{**} \exp \left(\left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) t + \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) \right) \quad (1.22)$$

Combining (1.20) and (1.22) equations gives

$$S_0^{(2)}(t) = \begin{cases} S_0^* \exp \left(\left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) t + \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) \right), t < 1 \\ S_0^{**} \exp \left(\left(\mu_2 - \frac{1}{2} \sigma_2^2 \right) t + \sigma_2 \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right) \right), t \geq 1 \end{cases} \quad (1.23)$$

Using (1.13) and (1.23) gives the complete summary of the coupled system of equations below:

$$S_0^{(1)}(t) = \begin{cases} S'_0 \exp\left(\left(\mu_1 - \frac{1}{2}\sigma_1^2\right)t + \sigma_1 dW_t^{(1)}\right), t < 1 \\ S''_0 \exp\left(\left(\mu_1 - \frac{1}{2}\sigma_1^2\right)t + \sigma_1 dW_t^{(1)}\right), t \geq 1 \end{cases}$$

$$S_0^{(2)}(t) = \begin{cases} S_0^* \exp\left(\left(\mu_2 - \frac{1}{2}\sigma_2^2\right)t + \sigma_2\left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)}\right)\right), t < 1 \\ S_0^{**} \exp\left(\left(\mu_2 - \frac{1}{2}\sigma_2^2\right)t + \sigma_2\left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)}\right)\right), t \geq 1 \end{cases} \quad (1.25)$$

2.1.3 Regime-Switching GBM-Access and Fidelity Assumptions

1. $S_t^{(1)}, S_t^{(2)} > 0$ a.s
2. $\mu_1, \mu_2, \sigma_1, \sigma_2 > 0$ constants, correlation coefficient $\rho \in (-1, 1)$ constant.
3. $S_0^{(i)}(t)$ is piecewise constant. $S_0^{(i)}(t) = S_0, W_t^{(1)}, W_t^{(2)}$ independent Brownian Motions.
4. Define $dW_t^{(2)} = \rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)}$
5. No arbitrage.

Theorem 1.2:(Existence and Uniqueness of the solution)

Therefore, we model the stock.

$$dS_t^{(1)} = \mu_1 S_t^{(1)} dt + \sigma_1 S_t^{(1)} dW_t^{(1)}$$

$$dS_t^{(2)} = \mu_2 S_t^{(2)} dt + \sigma_2 S_t^{(2)} dW_t^{(2)} \left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)} \right)$$

with piecewise $S_0(t)$ has unique strong solution.

$$S_t^{(1)}(t) = S'_0 \exp\left(\left(\mu_1 - \frac{1}{2}\sigma_1^2\right)t + \sigma_1 dW_t^{(1)}\right)$$

$$S_t^{(2)} = S_0^*(t) \exp\left(\left(\mu_2 - \frac{1}{2}\sigma_2^2\right)t + \sigma_2\left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)}\right)\right)$$

Proof: On each interval $[0, t]$ and $[t, T]$ the SDE has constant coefficients.

At $t = t$, we define $S_t^{(i)} = S_0, 2^{(i)}$ which ensures continuity of the solution in distribution. For the vector

$S_t = (S_t^{(1)}, S_t^{(2)})^T$, the drift vector $a(t, s)$ and diffusion matrix $b(t, s)$ satisfy:

$$\|a(t, s) - a(t, y)\| + \|b(t, x) - b(t, y)\| \leq k \|x - y\|. \quad (1.24)$$

Thus global Lipschitz holds. By multidimensional SDE theorem, unique strong solution exists.

3.1 Results And Discussion

Analysis of Model (1.1-1.2): Regime-Switching Initial-Condition Model

For the two-bank regime model, the analytical forms are taken as

$$S_t^{(1)} = S_0^{(1)}(t) \exp\left[\left(\mu_1 - \frac{1}{2}\sigma_1^2\right)t + \sigma_1 W_t^{(1)}\right],$$

$$S_t^{(2)} = S_0^{(2)}(t) \exp\left[\left(\mu_2 - \frac{1}{2}\sigma_2^2\right)t + \sigma_2\left(\rho W_t^{(1)} + \sqrt{1-\rho^2} W_t^{(2)}\right)\right] \quad (1.2)$$

where the deterministic level changes from 18 to 27 at $t = 1$, and $(\mu_1, \sigma_1) = (0.14, 0.28)$, $(\mu_2, \sigma_2) = (0.10, 0.35)$, $\rho = 0.6$. The correlated Brownian driver for the second asset is

$$\widetilde{W}_t^{(2)} = \rho W_t^{(1)} + \sqrt{1-\rho^2} W_t^{(2)}.$$

Since $W^{(1)}$ and $W^{(2)}$ are independent standard Brownian motions, $W_t^{(2)}$ is also a standard Brownian motion and satisfies

$$\text{Corr}(W_t^{(1)}, \widetilde{W}_t^{(2)}) = \rho.$$

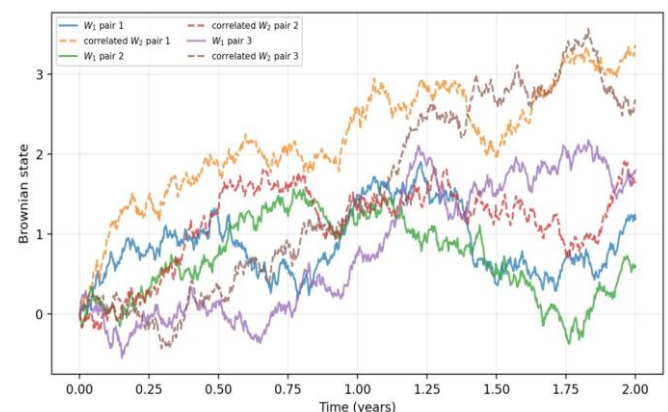


Figure 1: Correlated Brownian drivers Model.

Figure 1 shows that the second stochastic driver partially follows the direction of the first while retaining an independent component. Thus, $\rho = 0.6$ imposes positive co-movement between the stochastic innovations of the two bank-price processes. The Access Bank, 3 Sample Paths - Regime-Switch. Note: Jump at $t=1$ is common to all paths.

Post-shock, paths diverge. Illustrates policy impact and idiosyncratic risk. This situation simply means when post-shock sample paths diverge, it means that although both Access Bank and Fidelity Bank experienced the same policy jump at time t , their subsequent evolution is no longer tied together. The common policy shock raises the deterministic level for both, but the Brownian terms drive them apart over time. In terms of future share price: Access and Fidelity will have the same immediate boost from the CBN intervention, but beyond that, firm-specific performance, management decisions, and investor sentiment become the main drivers. So the gap between Access and Fidelity prices widens, reflecting that policy alone cannot guarantee identical long-term outcomes.

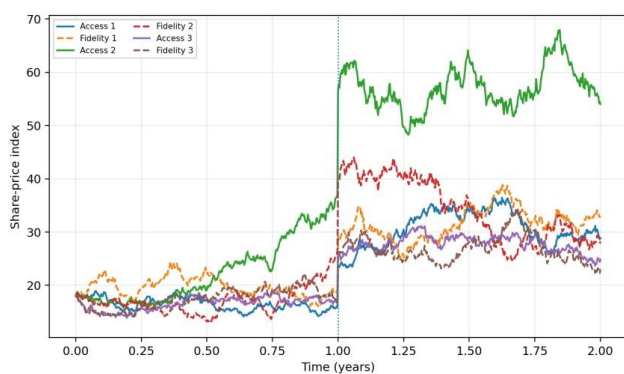


Figure 2: Sample Paths for the Two-Bank Regime Model

Figure 2 exhibits a visible level discontinuity around $t = 1$. This jump is not generated endogenously by the diffusion; it comes from the imposed change in the deterministic level from 18 to 27. The result may be interpreted as an intervention-type recapitalization shock. The sample paths for the two-bank regime model show an instantaneous jump because the deterministic initial condition $S_0^{(i)}(t)$ shifts from $S_{0,1}^{(i)}(t)$ to $S_{0,2}^{(i)}(t)$ at t . For Access and Fidelity, this jump from 18 to 27 represents a CBN-mandated recapitalization or policy intervention that instantly revalues both banks. It is not driven by market trading but by an external regulatory announcement: the market immediately prices in stronger capital buffers and reduced default risk, causing both share prices to gap up. However, because the jump is exogenous, the sustainability of the new price level depends on whether Access and Fidelity can generate earnings to justify the higher valuation post-shock.

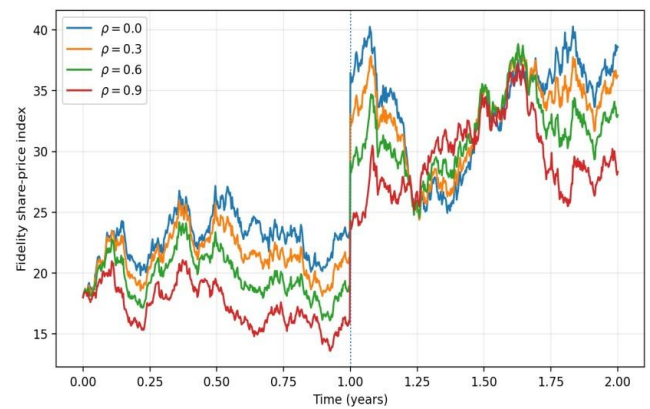


Figure 3: Sensitivity of Access and Fidelity Bank correlation coefficient.

The correlation sensitivity result in Figure 3 demonstrates that increasing ρ increases the extent to which the second bank responds to the same Brownian innovations as the first. This does not, by itself, change the marginal volatility parameter σ_2 ; it changes the dependence structure between the two processes. A value near zero produces weak instantaneous co-movement, while a value close to one makes the stochastic components move much more similarly. This scenario explains that an increase in the correlation parameter ρ means the Brownian Motions driving Access Bank and Fidelity Bank become more aligned. In practical terms, Access and Fidelity share prices will move together more strongly after the shock. If ρ is high, a negative market move drags both down together. This reduces the diversification benefit of holding both Access and Fidelity in the same systematic banking-sector risk. Thus, higher correlation implies that Access and Fidelity become more sensitive to common factors like CBN policy, interest rates, and macro conditions, and less driven by their own conditions, and less driven by their own idiosyncratic news.

4.1 Conclusion

The proof of existence and uniqueness on $[0, t)$ and $[t, T]$ ensures the regime-switching model is mathematically sound. Results show that the jump in $S_0(t)$ dominates short-term returns, while the stochastic term governs long-term dispersion. High correlation ρ implies limited diversification post-shock. The model supports “buy the rumor, sell the news” and mandates fundamental reassessment post-event. Therefore, regime-switching GBM is essential for capturing regulatory risk in emerging markets. We shall be considering deterministic and stochastic systems in valuation of assets in the next study.

References

- [1] Amadi, I. U. and Charles, A. (2022). An Analytical Method for the Assessment of asset price changes in capital market. *International journal of Applied science and Mathematical Theory*. vol.8(2) : 1-13.
- [2] Alvarez-Ramirez, J. and Cisneros, M.(2007). *A Dynamical Stochastic Coupled Model for Financial Markets*. Physica A: Statistical Mechanics and its Applications,376,333-342. Elsevier, Netherlands.
- [3]Shehzad, H.T., Anwar, M.A. and Razzaq, M (2023). *A Comparative Predicting stock prices using Heston and Geometric Brownian Motion Models*. Cornell University, 1-12, United States
- [4] Ugbebor, O. O., Onah, S. F. and Ojowo, O. (2001). An Empirical Stochastic Model of Stock Price Changes. *Journal of the Nigerian Mathematical Society*. Vol. 20: 95-101.
- [5] Amadi, I. U., Igbudu, R. and Azor, P. A. (2020). Stochastic Analysis of the Impact of Growth-Rates on Stock Market Prices in Nigeria. *Asian Journal of Economics, Business and Accounting*. Vol. 21 (24): 9-21.
- [6] Osu, B. O. and Amadi, I. U. (2022). A stochastic Analysis of stock Market price Fluctuation for capital market. *journal of applied Mathematics and computation*. Vol. 6(1):85-95.
- [7] Davis, I., Amadi, I. U. and Ndu, R. I. (2019). Stability Analysis of Stochastic Model for Stock Market Prices. *International Journal of Mathematics and Computational Methods*. Vol.4: 79-86.
- [8] Adeosun, M. E., Edeki, S. O. and Ugbebor, O. O. (2015). Stochastic Analysis of Stock Market Price Models: A Case Study of the Nigerian Stock Exchange (NSE). *WSEAS Transactions on Mathematics*. Vol. 14: 353-363.
- [9] Amadi, I. U. and Charles, A. (2022). Stochastic analysis of time -varying investment returns in capital market domain. *International Journal of mathematics and statistics studies*, Vol.10 (3): 28-38.
- [10] Ofomata, A. I. O., Inyama, S. C., Umana, R. A., and Omane, A. O. (2017). A stochastic model of the dynamics of stock price for forecasting. *Journal of Advance in Mathematics and computer science*, Vol.25 (6) :1-24.
- [11] Wokoma, D. S. A. (Bishop), Amadi, I. U. and Aboko, I. S. (2022). A Stochastic Model for Stock Market Price Variation. *Asian Research Journal of Mathematics*. Vol. 21 (4): 41-49.
- [12] Amadi, I. U., Ogbogbo, C. P. and Osu, B. O. (2022). Stochastic Analysis of Stock Price Changes as Markov Chain in Finite States. *Global Journal of Pure and Applied Sciences*. Vol. 28: 91-98.
- [13] Amadi, I.U and Okpoye , O.T (2022). Application of Stochastic Model in Estimation of Stock Return rates in Capital Market Investments. *International journal of Mathematical Analysis and Modelling*, Vol.5 issue 2, 1-14.
- [14] Okpoye, O.T. Amadi I.U. and Azor, P.A.(2023).An empirical Assessment of asset value function for capital market price changes. *International journal of Statistics and Applied Mathematics*, 8(3), 199-205.
- [15]Loko, O.P., Davies, I. and Amadi, I.U.(2023). The Impact of Fourier Series Expansion on the Analysis of Asset Value Function and its Return Rates for Capital Markets. *Asian Research Journal of Mathematics*, vol.19, issue 7,pp 76-91.
- [16] Amadi, I.U; Igbudu, R. and Azor, P.A.(2022). Stochastic Analysis of the impact of Growth-Rates on stock market prices in Nigeria. *Asian Journal of economics Business and Accounting* 21(24):9-21
- [17]Nwobi, F.N and Azor, P.A (2022).An Empirical Approach for the variation in Capital Market Price Changes. *International journal of Statistics and Mathematics*, Vol. 8(1), 164-172.
- [18] Osu, B.O., Eze, C.N. and Obi, C.N.(2020). The impact of stochastic volatility process on the values of assets. *Scientific Africa*, 9, e00513.
- [19] Lambert, D. and Lapeyre, B.(2007). *Introduction to Stochastic Calculus Applied to Finance*. CKC press.
- [20] George ,K.K. and Kenneth, K.L.(2019). Pricing a European Put Option by numerical methods. *International journal of scientific research publications*, 9, issue 11,2250-3153.